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Supporting Information

Broadband and Multiband Vibration Mitigation in Lattice Metamaterials with Sinusoidally-shaped Ligaments

Yanyu Chen¹, Feng Qian², Lei Zuo², Fabrizio Scarpa³, and Lifeng Wang^{1,*}

¹*Department of Mechanical Engineering
State University of New York at Stony Brook, Stony Brook, New York 11794, USA*

²*Department of Mechanical Engineering
Virginia Tech, Blacksburg, VA 24061, USA*

³*Advanced Composites Centre for Innovation and Science
University of Bristol, Bristol, BS8 1TR, UK*

Corresponding author: L. Wang, Email: Lifeng.wang@stonybrook.edu

S1. Experimental details

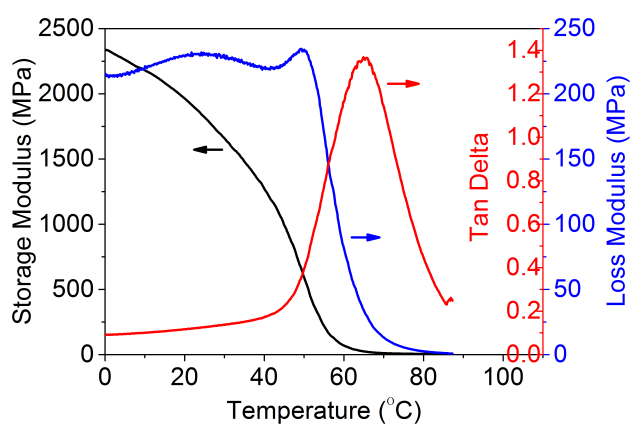
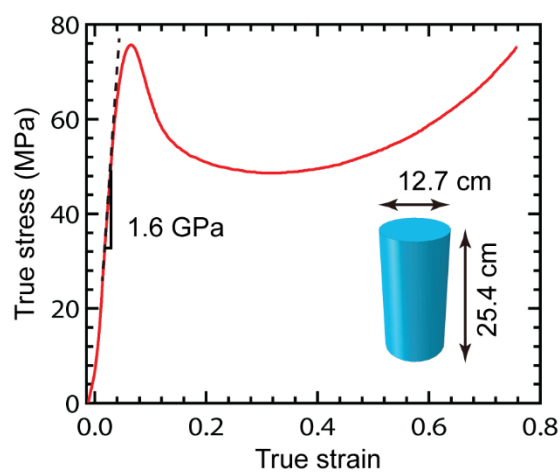
S1.1. Sample fabrication and characterization

All specimens including cylinders and lattice specimens used in the study are fabricated using an Objet260 Connex3 multi-material 3D printer (Stratasys, Ltd). Verowhite is the constitutive material used during the 3D printing. The chemistry of these materials is proprietary to Stratasys. The dimensions of the cylinder comply with the ones prescribed by the ASTM D695 standard.

The material properties of the Verowhite were obtained by measuring the mechanical response of the 3D printed cylinder specimens. Figure S1 shows the measured stress-strain curves (true and engineering strain) under uniaxial compression. According to ASTM D695, the basic properties of Verowhite are characterized by a Young's modulus $E=1.6 \pm 0.1$ GPa, Poisson's ratio $\nu=0.33$, and density of $\rho=1174$ kg m⁻³. Here the Young's modulus has been obtained from the measured stress-strain curve of the cylinder specimen. The density is obtained by averaging the densities of five cylinder specimens.

Most of polymer materials exhibit considerable damping effect. To study the effect of damping effect on the vibration mitigation capability of the proposed lattice metamaterials, dynamic mechanical analysis (DMA) was exploited to study and characterize Verowhite, including viscoelastic behavior, complex moduli and glass transition temperature of the materials. A sinusoidal stress were applied on the rectangular specimens printed by the 3D printer we use and the strain in the material was measured. The storage modulus and loss modulus are measured as a function of temperature, and the corresponding loss tangent is calculated. Combining loss tangent with stiffness, we can construct the damping figure of merits for Verowhite, as shown in Figure S2. It is noticable that at room temerpature (23 °C), the loss factor is about 0.12, which will be used in the frequency domain analysis.

Lattice metamaterials with different ligament wavelengths have an overall dimension of 229.5 mm×90 mm×25 mm and are composed of 5×2 unit cells (Fig. S3). The lattice metamaterials with different topologies have an overall dimension of 234 mm×160 mm×25 mm (Fig. S4). Two beams are also added to the top and bottom of the lattice structures to improve the alignment of the connection. Within the limitation of 3D printing technology, the layer orientation is found to influence the mechanical properties of the material; therefore, all the specimens are printed along the same orientation on the printer build platform. The as-fabricated specimens are kept at room temperature for 7 days to allow for the saturation of the curing.



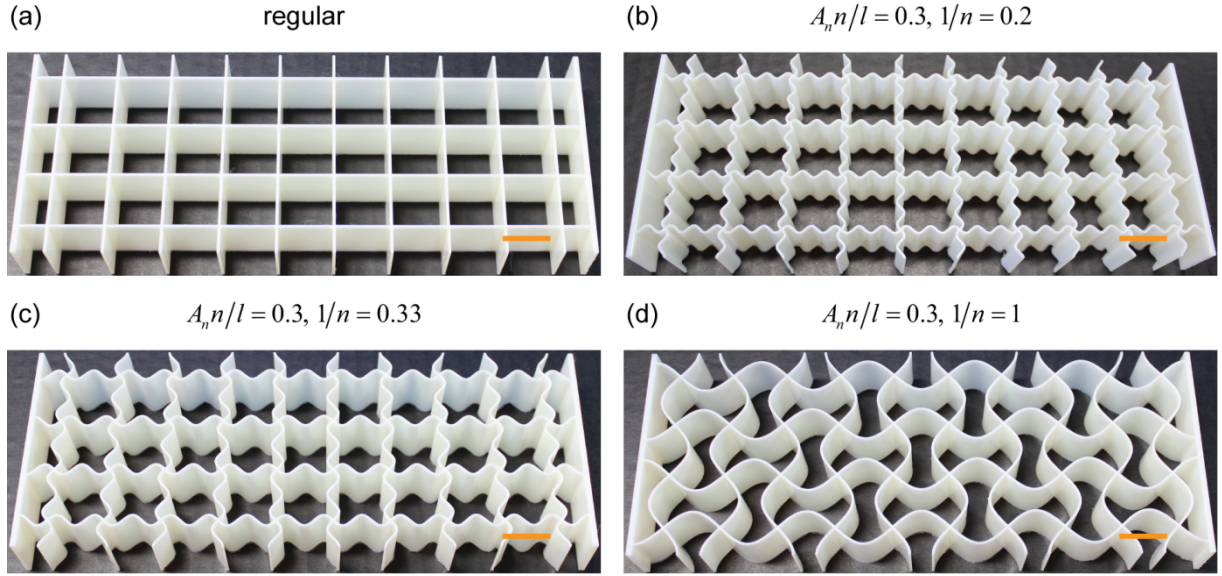


FIG. S3. 3D printed samples with different same ligament wave amplitude but different wavelengths. (a) regular lattice materials, (b) $A_n n/l = 0.30$, $1/n=0.2$, (c) $A_n n/l = 0.30$, $1/n = 0.33$, and (d) $A_n n/l = 0.30$, $1/n=1$. The out-of-the-plane thickness is 2.5 cm. Scale bar: 2 cm.

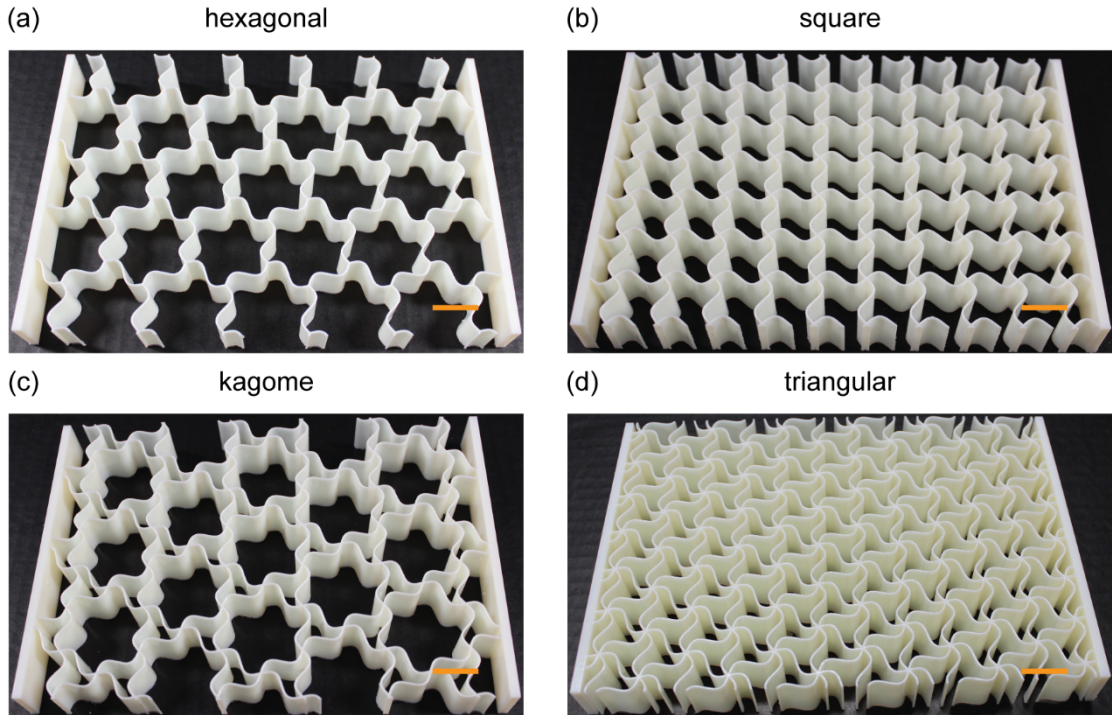


FIG. S4. 3D printed samples with different topologies. (a) Hexagonal, (b) square, (c) kagome, and (d) triangular. Here each ligament has a wave amplitude of $A_n n/l = 0.30$ and a wavelength of $1/n = 0.5$. The out-of-the-plane thickness is 2.5 cm. Scale bar: 2 cm.

S1.2. Wave transmission test

Low amplitude wave transmission tests are performed on the 3D-printed samples to validate the simulated phononic band gaps. A photograph and schematic diagram of the experimental setup is given in Fig. 3 (b). The sample is supported by two specially designed holders to avoid the friction damping effect from the testing table. An ICP[®] Impact Hammer with a hard tip (Model 086E80) is used to provide impulse forces exerted at the input end of the samples. The hammer can generate an impulse force with the frequency range up to 30 kHz that is sufficient to cover the frequency range of interest. A piezoelectric accelerometer is attached at the receiving end of the sample to capture the response signal transmitted from the input excitation. A Coco 80 data collector is adopted to record both the input force and output acceleration.

S2. Simulation details

S2.1. Dispersion relations of infinite periodic structures

S2.2.1 Construction of the first irreducible Brillouin zone

The periodicity of the proposed lattice metamaterials with a square topology can be described by two lattice vectors \mathbf{a}_1 and \mathbf{a}_2 (Fig. S5 (a)). The reciprocal lattice primitive vectors can be calculated as:

$$\mathbf{b}_1 = 2\pi \frac{\mathbf{a}_2 \times \hat{\mathbf{z}}}{\mathbf{a}_1 \cdot (\mathbf{a}_2 \times \hat{\mathbf{z}})} \text{ and } \mathbf{b}_2 = 2\pi \frac{\hat{\mathbf{z}} \times \mathbf{a}_1}{\mathbf{a}_1 \cdot (\mathbf{a}_2 \times \hat{\mathbf{z}})}. \quad (\text{S1})$$

By connecting the perpendicular bisectors of the reciprocal lattice, the corresponding Brillouin zone can be constructed (Fig. S5 (b)). Due to the presence of the rotation and mirror symmetries of the unit cells, the first irreducible Brillouin zone (gray-shaded region) is considered in the phononic dispersion relations simulation. Following the same procedure, we can construct the first irreducible Brillouin zones for lattice metamaterials with hexagonal, kagome, and triangular topology.

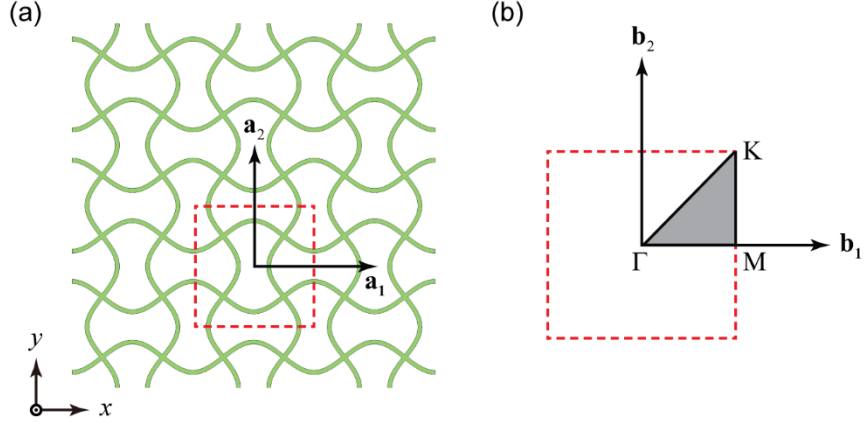


FIG. S5. Schematics of the unit cell for the proposed lattice metamaterials and the first irreducible Brillouin zone. (a) The proposed 2D lattice metamaterials with square topology described by two lattice vectors $\mathbf{a}_1 = [2l, 0]$ and $\mathbf{a}_2 = [0, 2l]$. The area surrounded by the dash lines is the representative unit cell. (b) Corresponding first irreducible Brillouin zone (gray-shaded region) in the reciprocal space.

S2.2.2 Bloch wave analysis for periodic lattice metamaterials

The governing equation of elastic wave propagating in the 2D lattice metamaterials is given by:

$$-\rho\omega^2\mathbf{u} = \frac{E}{2(1+\nu)}\nabla^2\mathbf{u} + \frac{E}{2(1+\nu)(1-2\nu)}\nabla(\nabla\cdot\mathbf{u}) \quad (\text{S2})$$

In Equation (S2) \mathbf{u} is the displacement vector and ω is the angular frequency. E , ν , and ρ are Young's modulus, the Poisson's ratio, and the density of the constituent material, respectively. The phononic dispersion relations are constructed by performing eigenfrequency analysis. The Bloch's periodic boundary conditions are applied at the boundaries of the unit cell such that:

$$\mathbf{u}_i(\mathbf{r} + \mathbf{a}) = e^{i\mathbf{k}\cdot\mathbf{a}}\mathbf{u}_i(\mathbf{r}) \quad (\text{S3})$$

where \mathbf{r} is the location vector, \mathbf{a} is the lattice translation vector, and \mathbf{k} is the wave vector.

The governing equation (S2) combined with the boundary condition (S3) leads to the standard eigenvalue problem:

$$(\mathbf{K} - \omega^2 \mathbf{M})\mathbf{U} = 0 \quad (\text{S4})$$

where \mathbf{U} is the assembled displacement vector, and \mathbf{K} and \mathbf{M} are the global stiffness and mass matrices assembled using standard finite element analysis procedure. The unit cell is discretized using 6-node triangular elements. In our simulations, we have used a discretization of 40 elements for the minimum wavelength. Equation (S4) is then numerically solved by imposing the two components of the wave vectors and hence calculates the corresponding eigenfrequencies. The phonon dispersion relations are obtained by scanning the wave vectors in the first irreducible Brillouin zone.

S2.2. Frequency domain analysis for transmission spectrum of finite-size structures

The transmission spectrum of the proposed lattice metamaterial was calculated by performing a frequency domain analysis. To model the elastic wave incident normally to the left-hand-side surface of the lattice metamaterials, a harmonic force with a small amplitude (1 N) is applied (Fig. 5 (a)). Perfectly matched layers (PMLs) are applied at the two ends of the homogeneous parts to prevent reflections by the scattering waves from the domain boundaries. The frequency range for the transmission simulation is 0~ 30 kHz. Here we have used a discretization 40 elements for the minimum wavelength. For the purpose of fair comparison, the simulated transmission is defined as the ratio of output acceleration amplitude to the input force amplitude. Effects of geometric features of sinusoidally-shaped ligaments, damping effect of 3D printed material, and topologies were investigated.